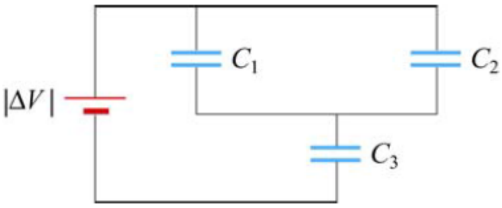
Question bank three

Capacitors and dielectrics

Question 1.

Find the equivalent capacitance for the combination of capacitors shown in Figure below

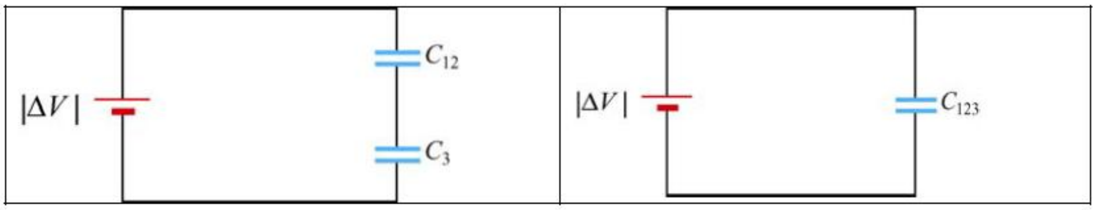


Capacitor connected in series and parallel (mixed)

Solution:

Since C_1 and C_2 are connected in parallel, their equivalent capacitance C_{12} is given by

$C_{12} = C_1 + C_2$



Now capacitor C_{12} is in series with C_3, So, the equivalent capacitance C_{123} is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{123} = \frac{C_{12}\, C_3}{C_{12} + C_3} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}$$

Question 2 a parallel plate capacitor of capacitance 4μF carries a charge of 600 μC. What is the potential difference between the plates of the capacitor?

Solution:

$\Delta \boldsymbol{V} = \frac{\mathrm{Q}}{\mathrm{C}}$ THUS

$\Delta \boldsymbol{V} = \frac{\mathbf{600}\mu\mathrm{C}}{4\ \mu\mathrm{F}} = \mathbf{15}\ \boldsymbol{V}$

Question 3 a parallel plate capacitor has an area of 40 cm* 50 cm and plates separated by 1cm.

a. What is the capacitance of this capacitor in unit of farad?
b. If it has a charge of $3.6*10^{-3}$ C, what is the potential difference across the capacitor?

Solution:

a. $\boldsymbol{C} = \frac{\boldsymbol{\varepsilon_0 A}}{\boldsymbol{d}}$

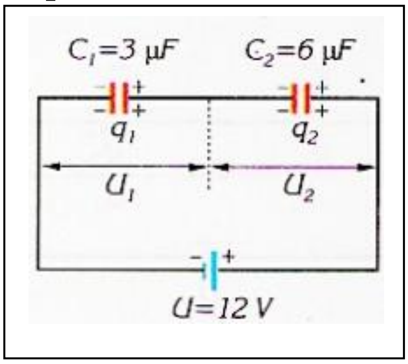
$$\boldsymbol{C} = \frac{\left(\mathbf{8.85} * \mathbf{10^{-12}} \frac{\boldsymbol{F}}{\boldsymbol{m}}\right)\left((\mathbf{0.4}\boldsymbol{m})(\mathbf{0.5}\boldsymbol{m})\right)}{\mathbf{0.01}\boldsymbol{m}} = \mathbf{18} * \mathbf{10^{-11}}\boldsymbol{F}$$

$$= \mathbf{180}\boldsymbol{pF}$$

b. $\Delta \boldsymbol{V} = \frac{\mathrm{Q}}{\mathrm{C}} = \frac{3.6*10^{-3}\mathrm{C}}{18*10^{-11}\ \mathrm{F}} = 2 * 10^{7}$ Volts

Question 4 two capacitors C_1 and C_2 are connected in series, find:

a. The equivalent capacitance b. the total charge c. the charge on each capacitor and d. the voltage across each capacitor**.**



Solution:

a. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$, $C_{eq} = 2\mu F$

b. $Q_t = C_{eq}\ \Delta \boldsymbol{V} = (2\mu F)(12V) = 24\mu C$

c. Since the charge on each capacitor in a series circuit is equal to the total charge,

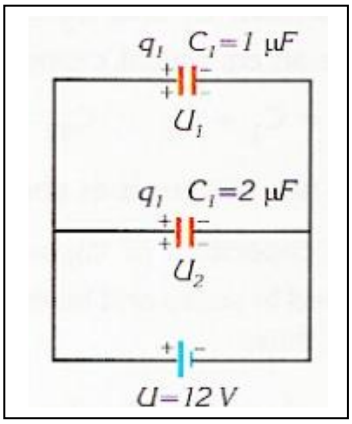
$$Q_t = Q_1 = Q_2$$

d. $Q_1 = C_1\ \Delta \boldsymbol{V}_1,\ \ 24\mu C = 2\mu F\ \Delta \boldsymbol{V}_1 \ \ \rightarrow\rightarrow \Delta \boldsymbol{V}_1 = 8V$

$Q_2 = C_2\ \Delta \boldsymbol{V}_2,\ \ 24\mu C = 6\mu F\ \Delta \boldsymbol{V}_2 \rightarrow\rightarrow \Delta \boldsymbol{V}_2 = 4V$

Question 5 two capacitors $C_1 = 1\mu F$ and $C_2 = 2\mu F$ are connected in parallel, find:

a. The equivalent capacitance b. the total charge c. potential difference across each capacitor and d. the charge on each capacitor.



Solution:

a. $C_{eq} = C_1 + C_2$

$C_{eq} = 1 + 2 = 3\mu F$

b. $Q_t = C_{eq}\ \Delta \boldsymbol{V}$

$Q_t = (3\mu F)(12V) = 36\mu C$

c. In parallel connection the potential difference across each capacitor is equal to the total potential difference of the circuit. So,

$$\Delta \boldsymbol{V}_t = \Delta \boldsymbol{V}_1 = \Delta \boldsymbol{V}_2 = 12V$$

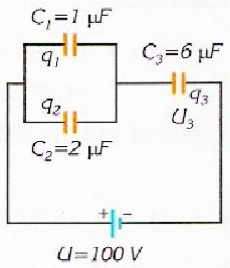
d. $Q = C\ \Delta \boldsymbol{V}$

$Q_1 = C_1\ \Delta \boldsymbol{V}_1 = 1 * 12 = 12\mu C$ $\qquad Q_2 = C_2\ \Delta \boldsymbol{V}_2 = 2 * 12 = 24\mu C$

$$Q_t = Q_1 + Q_2 = 12 + 24 = 36\ \mu C$$

Question 6

Three capacitors of capacitance $C_1=1\ \mu F$, $C_2=2\ \mu F$ and $C_3=6\ \mu F$ are connected as shown in the figure, and charged by a 100 V battery. Find:



a) The equivalent capacitance of the system.

b) The total charge of the system.

c) The potential difference, U_3.

d) The charge q_1.

Solution:

a) C_1 and C_2 are connected in parallel and C_3 is connected in series.

A system that consists of capacitors connected in series and in parallel, as in this example is called a complicated combination. In complicated combinations in order to find the equivalent capacitance we use the rules for series and parallel combinations.

Let us find the equivalent capacitance of the system.

Capacitors C_1 and C_2 are connected in parallel and have an equivalent capacitance C'.

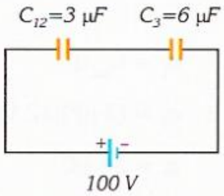
$$C_{12} = C_1 + C_2 \qquad C_{12} = 1 + 2 \qquad C_{12} = 3\ \mu F$$

The new system is as shown in the figure.

The capacitors of capacitance C_{12} and C_3 are connected in series and have an equivalent capacitance of C_{eq}, thus,

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$C_{eq} = 2\ \mu F$$



b) The total charge of the system is

$$q_t = C_{eq}\, U = (2\ \mu F)(100\ V) \qquad q_t = 200\ \mu C$$

c) Since charges on capacitors connected in series are equal, the charge on capacitor C_{12} and C_3 is equal to the total charge of the system, which is $q_t = 200\ \mu C$.

Therefore, from

$$q_3 = C_3\, U_3 \qquad 200\ \mu C = 6\ \mu F\ U_3$$

Thus, $U_3 = \dfrac{200\ \mu C}{6\ \mu C} = \dfrac{100}{3}\ V$

d) The charge q_1 on C_1 is,

$$q_1 = C_1\, U_1$$

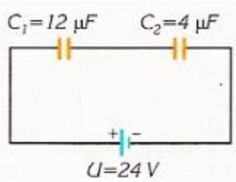
and $U_1 = U_2 = 100 - \dfrac{100}{3} = \dfrac{200}{3}\ V$

Therefore, $q_1 = (1\ \mu F)\left(\dfrac{200}{3}\ V\right)$

$$q_1 = \frac{200}{3}\ \mu C$$

Question 7

Two capacitors C_1 and C_2 are connected to the terminals of a battery, as shown in the figure. Find the energy stored in capacitor C_1.



Solution

$$W = \frac{1}{2} q_1 U_1 = \frac{1}{2}\frac{q_1^2}{C_1}$$

$$q_1 = q_2 = q_t$$

$$q_t = C_{eq}\, U$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12\ \mu F} + \frac{1}{4\ \mu F}$$

$$C_{eq} = 3\ \mu F$$

$$q_t = (3\ \mu F)(24\ V) = 72\ \mu C$$

$$q_1 = q_t = 72\ \mu C$$

$$W = \frac{1}{2}\frac{q_1^2}{C_1} = \frac{1}{2}\frac{(72\times10^{-6}\ C)^2}{(12\times10^{-6}\ F)}$$

$$W = 2.16\times10^{-4}\ J$$